A Method for Designing Three-Dimensional Configurations with Prescribed Skin Friction

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Nomenclature

c_f = skin friction coefficient
U, W = local boundary-layer velocity components in the x and z directions, respectively
δ = increment during iterations

Subscripts

e = local boundary-layer edge conditions
x = direction of local chord
z = local spanwise direction
o = values from previous iteration
∞ = freestream

Introduction

THIS note addresses the viscous design problem for three dimensional geometries Close to the body surface the flow is treated as a boundary layer The design is created as follows The skin friction distribution is the target, used as input in an inverse boundary-layer calculation The resulting pressure distribution is used as the input for an inverse potential flow calculation that generates the surface geometry.

The method described is based on inverse boundary layer calculations The input is the skin friction coefficient and the output the pressure distribution. Solutions of the three dimensional boundary-layer equations in the inverse mode have appeared recently in the literature [1-3] The solutions described herein are based on the method described in Ref 2 Anisotropic eddy viscosity is used for closure.

Inverse Boundary-Layer Scheme

The boundary layer scheme used is a modification to the one reported in Ref 2; thus, only the modification is described

If c_{fx} and c_{fz} denote the local vector skin friction components in the x and z directions, respectively, and c^*_{fx} and c^*_{fz} the target values, a Taylor expansion gives the following relations:

$$c_{fx} = c_{fx} + \frac{\partial c_{fx}}{\partial U_e}\Delta U_e + \frac{\partial c_{fx}}{\partial W_e}\Delta W_e \tag{1a}$$

$$c_{fz} = c_{fz} + \frac{\partial c_{fz}}{\partial U_e}\Delta U_e + \frac{\partial c_{fz}}{\partial W_e}\Delta W_e \tag{1b}$$

Equations (1) assume some proximity of the local values of the skin friction with the target values In practice this proximity is obtained by using the velocity components of the previous location as initial guesses From Eqs (1) one can solve for ΔU_e and ΔW_e and obtain

$$\Delta U_e = \left[(c_{fx}-c_{fx})\frac{\partial c_{fz}}{\partial W_e} - (c_{fz}-c_{fz})\frac{\partial c_{fx}}{\partial W_e}\right] \Big/ \left[\frac{\partial c_{fx}}{\partial U_e} \times \frac{\partial c_{fz}}{\partial W_e} - \frac{\partial c_{fx}}{\partial W_e}\frac{\partial c_{fz}}{\partial U_e}\right] \tag{2a}$$

$$\Delta W_e = \left[(c_{fz}-c_{fz})\frac{\partial c_{fx}}{\partial U_e} - (c_{fx}-c_{fx})\frac{\partial c_{fz}}{\partial U_e}\right] \Big/ \left[\frac{\partial c_{fx}}{\partial U_e}\frac{\partial c_{fz}}{\partial W_e} - \frac{\partial c_{fx}}{\partial W_e}\frac{\partial c_{fz}}{\partial U_e}\right] \tag{2b}$$

Equations (2) are used to obtain the increments in the components of the freestream velocity during the iteration procedure This method of computing the inverse boundary layer amounts to solving the direct problem iteratively until the desired boundary condition is achieved It was decided not to use the skin friction directly as a boundary condition, since that would have resulted in solving the boundary-layer equations with all conditions at the wall

Numerical Procedure

The inverse potential flow method used is similar to the procedure described in Ref 4 The procedure can be used to design wing alone configurations at subsonic and transonic speeds Several applications of this procedure are given in Ref. 4

The partial derivatives of c_{fx} and c_{fz} in Eqs (1) were evaluated using increments δU_e and δW_e with δ controlled from input. Convergence was judged by the percentage changes in both velocities (U_e, W_e) and skin friction coefficients (c_{fx}, c_{fz}) The boundary layer equations then were both solved in the direct mode, three times per iteration, and $\partial c_{fx}/\partial U_e$ $\partial c_{fx}/\partial W_e$ $\partial c_{fz}/\partial U_e$ $\partial c_{fz}/\partial W_e$ were computed using a first order finite difference approximation For

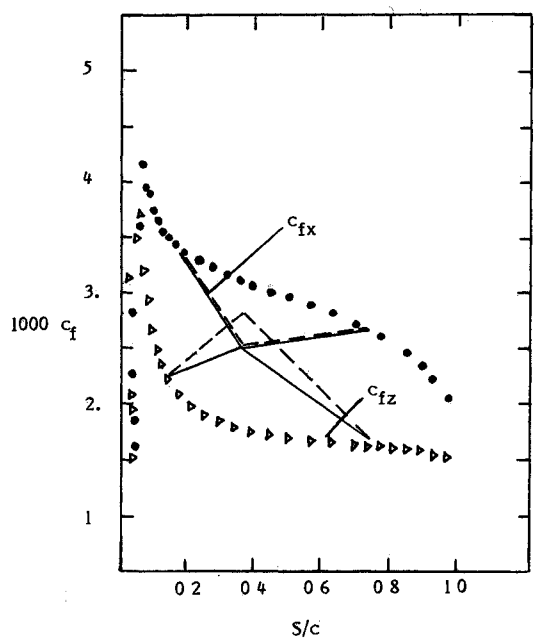

Fig 1 Original (● ▽) and imposed target (—, – – –) distributions of the skin friction.

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example, if U_{e0} and W_{e0} were the current values of the freestream velocity components, and c_{fx1} and c_{fz1} the current values of the skin friction, a new calculation was performed with $U_{e0}(1+\delta)$ and W_{e0} as the freestream velocity com ponents This resulted in new c_{fx2} and c_{fz2} Then

$$\frac{\partial c_{fx}}{\partial U_e} = \frac{c_{fx2} - c_{fx1}}{\delta U_{e0}} \tag{3}$$

$$\frac{\partial c_{fz}}{\partial U_e} = \frac{c_{fz2} - c_{fz1}}{\delta U_{e0}} \tag{4}$$

The same procedure was used for the $\partial c_f / \partial W_e$ derivatives and the velocities were updated using Eqs (2) and (3) as follows:

$$U_e = U_{e0} + \Delta U_e \tag{5}$$

$$W_e = U_{e0} + \Delta W_e \tag{6}$$

In Ref 2, where the input displacement thickness was translated into a boundary condition for the inverse calculations, upwind differencing of the convection terms was found necessary in order to obtain smooth solutions In this work where the input skin friction was approached iteratively, there was no need for upwinding This may be due to the fact that the input boundary condition was achieved only within a prescribed tolerance

Discussion of Results

A large aspect ratio transport wing was selected as the basic configuration The wing had a sweep of 25 deg at the quarter chord, an aspect ratio of 8, and a taper ratio of 0 4 The calculations were made for a unit Reynolds number of 10^6 The streamwise length of the wing root was 5 78 units long The freestream Mach number used (0 15) makes the error due to the incompressible flow assumptions in the boundary layer very small

For reasons of simplicity, and because the procedure does not include wake calculations, the target skin friction was prescribed only on part of the upper surface of the wing. The leading- and trailing edge regions were excluded The boundary layer was solved on all 22 span stations, and the solution points on the surface were chosen to coincide with the locations where the solution is provided from the potential flow code The local sweep was used, and for a wing with the characteristics mentioned previously, the infinite swept wing assumptions were reasonable to make The surface coordinate S is measured from the leading edge line and on the airfoil normal to the leading edge Since the leading and trailing edge regions were excluded, the curvature terms were not included in the boundary layer calculations through the metrics The length c is the local chord length of the airfoil normal to the leading edge

The boundary layer was tripped at a fixed point after the leading edge at approximately the location of the peak suction close to $S/c = 0\ 05$ In the inverse mode the calculations started at a fixed streamwise point close to $S/c = 0\ 16$ The direct mode was used up to that point The isotropic eddy viscosity was used in all the cases

The design was performed as follows Excluding a region around the midchord of the upper surface, the skin friction values, and, therefore, the pressure, were kept unchanged and equal to the values computed from the direct mode on the wing Figure 1 shows the imposed linear variation of c_{fx} and c_{fz} It raises the c_{fz} and lowers the c_{fx} values in the region approximately $0\ 16 < S/c < 0\ 74$ on the upper surface As expected the computed components U_e and W_e of the local freestream velocity follow the trend of the skin friction values and are shown in Fig 2 This imposed rotation of the skin friction vector in the outboard spanwise direction may be

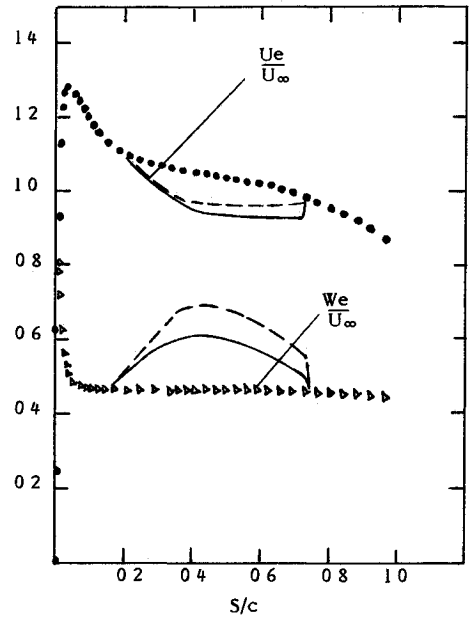

Fig 2 Original (● , ▽) and resulting (—, – – –) edge velocity com ponents

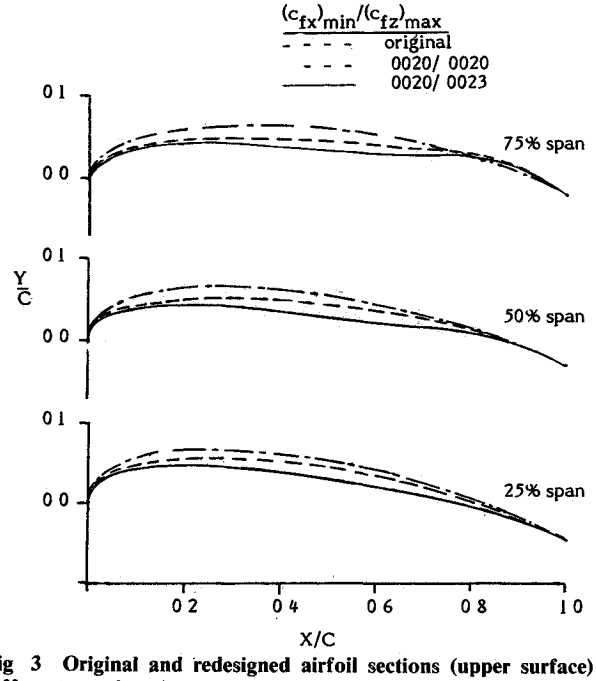

Fig 3 Original and redesigned airfoil sections (upper surface) at different span locations.

accomplished without excessive thickening of the boundary layer This turning of the skin friction vector reduces its drag producing component Note that for two dimensional or axisymmetric bodies this mechanism of drag reduction does not work While prescribing target skin friction, care must be taken to avoid excessive turning of the skin friction vector, leading to separation

The pressure distributions show the effect of the imposed skin friction Because no effort was made to prescribe a target skin friction so that the new and original pressures were joined smoothly the following behavior is observed A discontinuity in the pressure distribution is generated at approximately $S/c \simeq 0\ 74$ The inverse potential code pre dicted a somewhat smoother pressure distribution as it should, and produced airfoils with an abrupt change of profile slope at that location

The upper surface of the redesigned wing sections at the 25 50, and 75% span locations are shown in Fig 3 There is a flattening of the upper surface similar to that observed in high lift airfoils [7] As previously mentioned, the sudden ac celeration at $S/c \simeq 0$ 74 produced a discontinuity of the profile slope at that location It was not possible to impose the target skin friction, shown in Fig 2, on the whole upper surface of the wing When this was done, the design pressures failed to match the target close to the wing tip Therefore, starting at span station 17, and up to the last station (22) at the wing tip the design skin friction was the linear interpolation between the values at station 17 and the original values at the tip This procedure generated design pressures close to the target for the entire wing

Conclusion

The results presented are a very small sample of the possible choices; however, they demonstrate a viable procedure for solving the viscous design problem in three dimensional flow Designs with strong viscous/inviscid interaction effects have to be attempted using the present method after the two inverse codes are coupled It can be concluded that 1) the work extends ideas already applied in two-dimensional problems[5 7] to the three dimensional case, and 2) the work can be used to design wings with a specified target skin friction Therefore it can be used for designing for low drag, high lift, or avoidance of separation

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